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ESTIMATION OF MAINTENANCE COSTS OF A PIPELINE FOR A U-SHAPED HAZARD RATE FUNCTION IN THE IMPRECISE SETTING

ESTYMACJA KOSZTÓW EKSPLOATACYJNYCH RUROCIĄGU DLA U-KSZTAŁTNEJ FUNKCJI INTENSYWNOŚCI USZKODZEŃ PRZY NIEPRECYZYJNYM PODEJŚCIU

In this paper, we discuss imprecise settings for an evaluation of the maintenance costs of a water distribution system (WDS). Moments of failures of pipes are modelled using a newly proposed three-piece convex hazard rate function (HRF) for which number of previous failures is taken into account, too. Both fuzzy sets and shadowed sets are used to model the impreciseness of important parameters of this HRF and the costs of maintenance services. Contrary to more classical and widely-used approaches to cost analysis (i.e. a constant yield or nominal value of money), a strictly stochastic process (i.e. the one-factor Vasicek model) of an interest rate is assumed in the analysis of maintenance costs. This approach models future behaviour of the interest rate (i.e. the future value of money) in a more realistic way. Respective algorithms together with exemplary results of numerical simulations for two setups, which are related to fuzzy and shadowed sets, are also provided.

Keywords: *water distribution system, maintenance costs, convex hazard rate function, Monte Carlo simulations, fuzzy sets, shadowed sets.*

W niniejszym artykule omawiamy nieprecyzyjne podejścia do problemu obliczenia kosztów eksploatacji systemu dystrybucji wody (WDS). Czasy uszkodzeń rur modelowane są z wykorzystaniem nowo zaproponowanej trzyczęściowej wypukłej funkcji intensywności uszkodzeń (hazard rate function, HRF) dla której brana jest pod uwagę również liczba wcześniejszych uszkodzeń. Do modelowania nieprecyzyjności istotnych parametrów tej HRF oraz kosztów działań serwisowych są wykorzystywane zarówno zbiory rozmyte jak i zbiory cieniowane. W przeciwieństwie do bardziej klasycznych i szeroko wykorzystywanych podejść do analizy kosztów eksploatacji (tzn. stałej stopy procentowej lub wartości nominalnej pieniądza), założono ściśle stochastyczny proces (tzn. jednoczynnikowy model Vasicka) dla stopy procentowej. Podejście to modeluje przyszłe zachowanie stopy procentowej (czyli przyszłej wartości pieniądza) w bardziej realistyczny sposób. Zaprezentowano również odpowiednie algorytmy wraz z przykładowymi wynikami symulacji numerycznych dla dwóch zestawów parametrów, związanych ze zbiorami rozmytymi i cieniowanymi.

Słowa kluczowe: *system dystrybucji wody, koszty eksploatacji, wypukła funkcja intensywności uszkodzeń, symulacje Monte Carlo, zbiory rozmyte, zbiory cieniowane.*

1. Introduction

To deliver water of desirable quality and in necessary quantity, various maintenance services (like repairs and replacements of connections) for a water distribution system (WDS) are necessary. The literature devoted to different aspects of these problems, like modelling reliability of a WDS or a calculation of costs of the maintenance services, is abundant. We refer the reader to some detailed and interesting reviews, e.g., [16, 29, 30]. The articles related to the problem of maintenance of a WDS are really diversified, too. Some of them discuss hydraulic and physical characteristics of parts of a WDS (see, e.g., [5, 18]), other focus on a "macro-management" of a WDS (see, e.g., [22]) or its "micro-management" scale (see, e.g., [1]), or propose the application of artificial neuronal nets in a monitoring system (see, e.g., [23]) or even artificial intelligence (see, e.g., [24]).

In this paper, we discuss a simulation approach to the estimation of the maintenance costs related to repairs and replacements of pipes. Usually, to calculate these costs in an appropriate manner, a relatively long-time horizon has to be considered. Such a time interval covers 20, 50 or even 60 years (see, e.g., [12]). But in most of the papers, a constant rate for money flow or even nominal costs are considered, which is rather counter-intuitive, because one unit of money, which is paid now, and the same unit in 50-60 years, are not equal. Moreover, the assumption about a constant discount factor is too strong and unrealistic for the mentioned long time horizons. Therefore, a variable interest rate should be considered in a more real-life approach. Then, in this paper, for modelling such variable interest rates we apply the one-factor Vasicek model.

In the literature, many models of intensities of malfunctions of parts of a WDS have been proposed. Some of them are related to physical aspects of a pipe and are described by formulas (like the Hazen-Williams equation, see, e.g., [12]). Other models are based on Markov or semi-Markov processes (see, e.g., [13, 17, 25]) or are described using a hazard rate function (HRF, see, e.g., [30] for a comprehensive review). In this paper, we propose a new kind of HRF that describes three important stages of the "life" of a pipe and takes into account an increasing deterioration of a material of a pipe. The proposed HRF completely fulfils requirements formulated by some authors (see, e.g., [30]). It can be easily adapted to real-life data. Moreover, the respective generation algorithm is very efficient in the Monte Carlo simulations which are widely used in modelling complex systems (see, e.g., [3]).

This paper can be seen as further development of ideas proposed earlier in [25, 26], as some new concepts are also considered. Thus, our contribution to problems considered in this paper is five-fold.

First, a new, more general hazard rate function, which describes the intensities of malfunctions of pipes in a WDS, is introduced. Contrary to other HRFs, which were previously discussed in the literature, it has some appealing features. It is U-shaped and it models three important states of a connection: a starting burn-in period (immediately after a repair or an installation of a pipe, when the intensity of malfunctions is a decreasing function), a middle stable state (when initial problems with a connection have passed, so the level of failures is relatively low), and a later wear-out period (when the intensity of malfunctions is an increasing function, with higher values than during its stable state). This HRF also depends on number of previous repairs of the given pipeline, so an increasing deterioration, which is caused by recurring stresses related to repairs, can be taken into account. Moreover, a numerically efficient algorithm for the generation of random times of failures for this HRF is also provided. It leads us to a direct application of the Monte Carlo (MC) simulations to simulate the behaviour of the whole WDS. Contrary to [26], the newly proposed HRF has three, instead of only two states (i.e. it is U-shaped instead of its previous V-shaped version).

Second, apart from the mentioned HRF, we use a special random distribution with a decreasing intensity function and finite support to model times of services related to malfunctions of a WDS (i.e., repairs and replacements of pipes). A numerically efficient algorithm for the generation of random times of these services is provided, too. Therefore, the MC approach can be used to simulate the respective values of the mentioned times.

Third, almost all of the parameters of the model (apart from these related to the variable interest rate, an unconditional replacement age, and time horizon for the Monte Carlo simulations) are fuzzified to express our limited information about their real values. Moreover, we provide a general framework for using different kinds of fuzzy numbers (like triangular fuzzy numbers, trapezoidal fuzzy numbers or left-right fuzzy numbers) as the parameters of the model to calculate the present value of the maintenance costs or other important characteristics of a WDS. This framework can be then applied to other models of failure intensities for a WDS. Therefore, we generalize our previous considerations from [25] to a more general fuzzy approach including new types of fuzzy numbers.

Fourth, apart from the imprecise setting related to fuzzy numbers, we apply shadowed sets to describe the parameters of the model. An introduction of shadowed sets can be very fruitful because it enables us to both consider impreciseness (which is, e.g., related to an expert's opinion) and to dramatically limit amount of necessary numerical simulations to estimate the considered characteristic of the model (like the present value of the maintenance costs) in comparison with the previously mentioned fuzzy setting.

Fifth, apart from the theoretical framework for both fuzzy numbers and shadowed sets setups, we provide respective numerical algorithms together with examples of numerical simulations based on the Monte Carlo approach. Some important measures of a WDS, like the present value of the maintenance costs, are approximated using fuzzy numbers and shadowed sets. Therefore, apart from a "strict" value (related to the core of a fuzzy / shadowed set), an additional "imprecise" interval (related to the support of a fuzzy / shadowed set) is calculated.

It should be pointed out, that the stochastic model of the interest rate (i.e., the one factor-Vasicek model) is directly embedded into our Monte Carlo simulations, as in [26]. To our best knowledge, application of the variable interest rate is still a new idea, which is not considered in other papers. But there are significant differences in outputs (like estimated values of costs of a WDS, see, e.g., [25, 26]), between models with a constant yield and with a variable discount factor. Therefore the stochastic model of the interest rate should be preferred in an estimation of the maintenance costs.

This paper is organized as follows. In Section 2, a new type of a U-shaped hazard rate function, which describes times of the failures of a connection, is presented. A numerical algorithm for a respective density, which is based on this HRF, is also provided. In Section 3 and 4, a model of maintenance costs for a WDS based on constant and random variable costs of repairs and replacements, together with a new random distribution for modelling times of repairs and replacements, are discussed. Section 5 is devoted to two imprecise settings related to fuzzy numbers and shadowed sets. Respective algorithms for these setups are also provided there. We illustrate these two approaches in Section 6 with examples of numerical analysis using the Monte Carlo simulations. The paper is concluded in Section 7.

2. Properties of U-shaped HRF

Let us suppose, that the considered WDS is modelled by a graph of connections G. In this graph, each connection (i.e., a pipeline which is a part of this WDS) is represented as an edge, and possible sources or outflows are denoted by nodes. In the following, we focus only on the edges of the graph G, i.e., the connections of the WDS. Let us assume, that these connections behave in a statistically independent way, i.e., time of a malfunction of one pipe does not influence the quality and possible malfunctions of other pipes. In the literature, multistate systems with embedded dependencies of components are also considered (see, e.g., [2]).

We assume that times of failures for each connection are described by a hazard rate function (abbreviated further as HRF) $\lambda(x|n_r)$, which is given by the formula:

$$\lambda(x|n_r)=\begin{cases} -a_0x+b_0+\alpha_r n_r & if \quad x\in\left[0,x_0^*\right) \\ a_1x+b_1+\alpha_r n_r & if \quad x\in\left[x_0^*,x_1^*\right) \\ a_2x+b_2+\alpha_r n_r & if \quad x\geq x_1^* \end{cases}, \tag{1}$$

where $a_0>0, a_1>0, a_2>a_1, b_0\geq a_0x_0^*, x_0^*>0$, $x_1^*>0, \alpha_r>0$ and

$$b_1=-x_0^*\left(a_0+a_1\right)+b_0 \ , \ b_2=x_1^*\left(a_1-a_2\right)+b_1 \ .$$

As it is seen in Figure 1, this HRF is a U-shaped function with three linear segments, for which:

- $-a_0$ is a directional component of the descending linear part of this HRF (i.e., a left-hand side of this function, for which $x\in\left[0,x^*\right)$),
- a_1 is a directional component of the middle, ascending linear part (i.e., when $x\in\left[x_0^*,x_1^*\right)$),
- a_2 is a directional component of the right-hand side, ascending linear part (i.e., when $x\geq x_1^*$),
- b_0 is related to a vertical shift of the whole HRF,
- x_0^* and x_1^* are horizontal values of the points, where this HRF changes its behaviour,
- α_r is a parameter of deterioration, which is related to a single, previous malfunction of a connection,
- n_r is number of previous malfunctions of a connection, if there were repairs afterwards.

When a connection is replaced with a completely new component, then $n_r=0$ is set. It means that the previously mentioned parameter α_r reflects a level of fatigue related to prior malfunctions and

repairs (see also [26]). Such an influence of the number of previous malfunctions on the current condition of a connection is frequently postulated in the literature (see, e.g., [30]). In real-life applications the above-mentioned parameters of the HRF (1) should be properly adjusted to the existing data, because number of failures varies, e.g., in [6], the average rate of failures is reported as values from 2.3 up to 34.8 (per 100 miles of pipes per a year) depending on a material of the considered pipe.

The values x_0^* and x_1^* are connected with three important states of a connection (see also, e.g., [4]): its initial burn-in period (after a previous repair or just after an installation of a new pipe), a stable state (when possible initial problems have passed, so the level of failures is relatively low) and a wear-out period (when the intensity of failures increases with passing time, because of existing problems of "old age" of the connection).

This new HRF, which is given by (1), is a more complex, U-shaped function if it is compared to its previous V-shaped counterpart, which was introduced in [26]. Moreover, a new additional state (the stable state) can be modelled using (1). Then, this HRF can be used to describe the intensity of malfunctions, taking into account three completely different states of quality of a pipeline and progress of fatigue of a connection (which is related to the number of previous repairs n_r). Therefore, this HRF can be better adjusted to real-life data, if it is compared to other types of functions discussed in the literature. This HRF also meets the requirements concerning functions describing the intensity of malfunctions (see [30] for additional details).

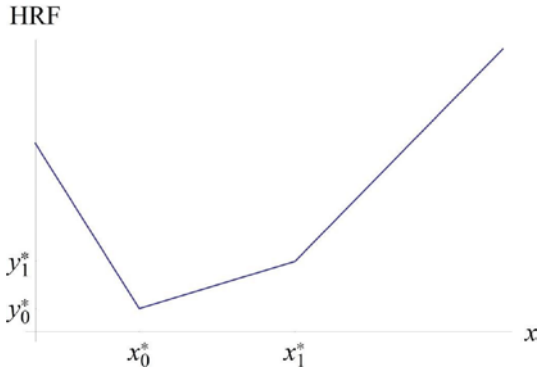


Fig. 1. Exemplary plot of the introduced HRF

To simplify further formulas, let us assume that:

$$b_0^{'} = b_0 + \alpha_r n_r \,,\; b_1^{'} = b_1 + \alpha_r n_r \,,\; b_2^{'} = b_2 + \alpha_r n_r \,.$$

Because:

$$\lambda(x) = \frac{f(x)}{R(x)} \,, \tag{2}$$

where $f(x)$ is a pdf (probability density function), $R(x) = 1 - F(x)$ and $F(x)$ is a cdf (cumulative density function), then in the case of (1), we get:

$$f(x) = \begin{cases} \left(-a_0\, x + b_0^{'}\right)\exp\left(\frac{1}{2}a_0\, x^2 - b_0^{'}\, x\right) & \text{if} \quad x \in \left[0, x_0^*\right) \\ \left(a_1\, x + b_1^{'}\right)\exp\left(-\frac{1}{2}a_1\, x^2 - b_1^{'}\, x + c_1\right) & \text{if} \quad x \in \left[x_0^*, x_1^*\right) \\ \left(a_2\, x + b_2^{'}\right)\exp\left(-\frac{1}{2}a_2\, x^2 - b_2^{'}\, x + c_2\right) & \text{if} \quad x \geq x_1^* \end{cases} \tag{3}$$

where:

$$c_1 = \frac{1}{2}\left(a_0 + a_1\right)\left(x_0^*\right)^2 + \left(-b_0^{'} + b_1^{'}\right)x_0^*,$$

$$c_2 = c_1 + \frac{1}{2}\left(-a_1 + a_2\right)\left(x_1^*\right)^2 + \left(-b_1^{'} + b_2^{'}\right)x_1^*$$

In Figure 2, an exemplary plot of the density (3) with the two previously mentioned points, where the HRF changes its behaviour ($x_0^* = 0.8, x_1^* = 3$, namely), is provided.

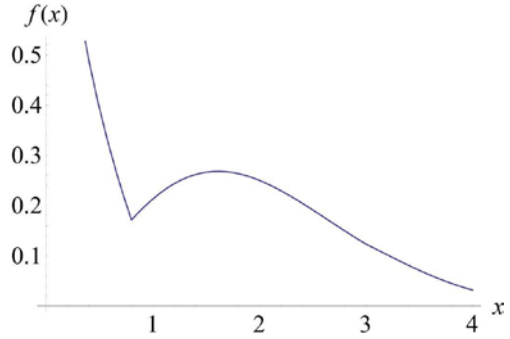


Fig. 2. Exemplary plot of the density for the introduced HRF

To simulate random times of malfunctions, it is necessary to provide a numerically efficient algorithm, which generates random variables based on (3). It can be done using the composition method and the inversion method (see, e.g., [28] for a necessary introduction). If the composition approach is applied, then a respective pdf $f(x)$ is given by:

$$f(x) = \sum_{i=1} f_i(x)\, p_i \,,$$

where $f_i(x) \geq 0$ is a density and $p_i \geq 0$ is a discrete probability for $i = 1, 2, \ldots$. In the case of (3), we have:

$$p_1 = P\left(X \in \left[0, x_0^*\right)\right) = 1 - \exp\left(\frac{1}{2}a_0\left(x_0^*\right)^2 - b_0^{'}\, x_0^*\right),$$

$$p_2 = p_2^{'}\, \exp\left(x_0^*\left(-b_0^{'} + \frac{1}{2}a_0 x_0^*\right)\right),$$

$$p_3 = 1 - p_1 - p_2 \,, \tag{4}$$

where

$$p_2^{'} = 1 - \exp\left(\left(x_0^* - x_1^*\right)\left(b_1^{'} + \frac{1}{2}a_1\left(x_0^* + x_1^*\right)\right)\right).$$

Using the inversion method, for $f_1(x)$ (if $x \in \left[0, x^*\right)$), $f_2(x)$ (when $x \in \left[x_0^*, x_1^*\right)$) and $f_3(x)$ (when $x \geq x_1^*$), the respective inversions of their cdfs (compare also with [26]) are given by

$$F_1^{-1}(y) = \frac{b_0^{'} - \sqrt{\left(b_0^{'}\right)^2 + 2a_0 \ln\left(1 - p_1 y\right)}}{a_0} \,,$$

$$F_2^{-1}(y)=\frac{-b_1^{'}+\sqrt{\left(b_1^{'}+a_1x_0\right)^2-2a_1\ln\left(1-p_2^{'}y\right)}}{a_1},$$

$$F_3^{-1}(y)=\frac{-b_2^{'}+\sqrt{\left(b_2^{'}+a_2x_1^{*}\right)^2-2a_2\ln(1-y)}}{a_2}. \quad (5)$$

Then, Algorithm 1 can be directly used to simulate times of the failures in a numerically efficient way.

Algorithm 1 (Generation procedure for the HRF)
Input: A set of the parameters of the HRF (1).
Output: A random time of a failure X.

Calculate p_1,p_2,p_3, *which are given by (4);*
Generate independent random values U,Y *from the uniform standard distribution* $U[0;1]$*;*

if $U\le p_1$
$X=F_1^{-1}(Y)$ *(see (5));*
else
if $U\le p_2$
$X=F_2^{-1}(Y)$ *(see (5));*
else
$X=F_3^{-1}(Y)$ *(see (5));*
return X

The expected value of the density (3) can be numerically computed, e.g., for $a_0=0.6,a_1=0.2,a_2=0.8,b_0=0.65,x_0^{*}=0.5,x_1^{*}=10,\alpha_r=0.1,n_r=0$ we get 4.53037 (about 4.5 years if time unit is assumed to be a year, see also values in Table 2) and when one malfunction has happened (i.e., for $n_r=1$) this value changes to 3.765 (about 3 and 3/4 years).

3. Model of maintenance times

We assume that each connection in time t can be in one of the following states: *working, under repair, under replacement*. We also assume that immediately after a failure, the respective connection is repaired or replaced by a new one, so there is no waiting time for a necessary service.

As it was assumed, working times WT_i (i.e., times between malfunctions) are *iid* random variables described by the density (3). In the following, repairing times RT_i (times, when a connection is repaired) and replacement times PT_i (times, when a connection is replaced with a new one) are also modelled by a new kind of a probability distribution.

A replacement of a pipe is related to a deterministic and unconditional replacement age P^{*}. If the current sum of working and repairing times for the considered connection is larger than P^{*}, i.e.,

$$\sum_{i=1}^{j}WT_i+RT_i>P^{*}, \quad (6)$$

then this connection is replaced with a new one (instead of one more repair). Afterwards, $n_r=0$ is set, so this replacement "clears" a deterioration process of the given pipeline.

To model the mentioned repairing and replacement times we apply a special distribution related to a decreasing linear intensity function, i.e.,

$$\lambda(x)=-cx+cd \quad if \quad x\in[0,d],$$

where $c>0,d>0$, and c is its directional component while d is a right-hand side limit of its support. Then, using (2), the respective density is equal to:

$$f(x)=\frac{1}{1-\exp\left(-\frac{cd^2}{2}\right)}c(d-x)\exp\left(c\left(\frac{x^2}{2}-dx\right)\right) \quad if \quad x\in[0,d], \quad (7)$$

An exemplary plot of this density can be seen in Fig. 3. To simulate random values from (7), the inversion method can be directly applied, so we get

$$F^{-1}(y)=d-\frac{1}{\sqrt{c}}\sqrt{cd^2+2\log\left(1+\left(\exp\left(-cd^2/2\right)-1\right)y\right)}. \quad (8)$$

Then, Algorithm 2 can be used to simulate values of the repairing and the replacement times. In the following, to distinguish parameters of these two types of services, we use c_R,d_R (in the case of the repairing times) or c_P,d_P (for the replacement times, respectively). The expected value of the density (7) can be numerically calculated, e.g., for $c=10,d=0.014$ we get 0.004666 (i.e., we have $\frac{5}{360}=0.013889,\frac{1}{360}=0.0027778$, so d is about 5 days for 360 days calendar, and the expected value is then about 1.68 day, see also values in Table 2).

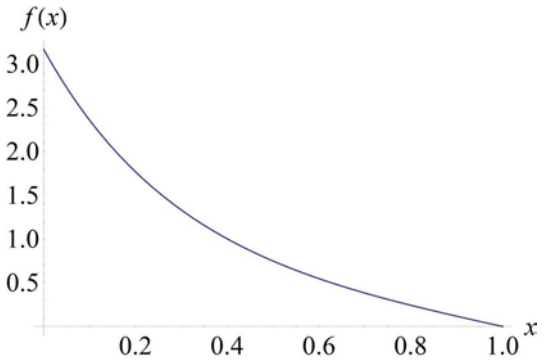


Fig. 3. Exemplary plot of the density for the repairing and the replacement times

Algorithm 2 (Generation procedure for the repairing / replacement times)

Input: Parameters c_R,d_R or c_P,d_P.
Output: A random repairing / replacement time X.

Generate independent random value U *from the uniform standard distribution* $U[0;1]$*;*

$X=F^{-1}(Y)$ *(see (7));*
return X

To simulate the whole WDS, we assume that its connections behave in a statistically independent way. Then, using the MC approach, the random times of malfunctions $t_1, t_2, \ldots$ (which are directly related to the working times), together with the repairing and the replacement times (i.e., RT_i and PT_i), can be generated.

4. Model of maintenance costs

In the following, the costs of repairs and replacements are estimated using the Monte Carlo (MC) simulations. The Monte Carlo approach is applicable if, apart from the considered HRF (1), some numerically feasible probability distributions for the repairing times RT_i and the replacement times PT_i are also used, like the introduced density (7). Because of the MC simulations, there is no need to solve complex theoretical formulas to take into account the condition (5).

In this paper, we focus only on the maintenance costs related to the replacements and the repairs. Other types of costs (like costs of water losses, loss of revenues, etc. – see, e.g., [4, 12, 19]) are commonly considered in the literature. Among others, restoration and diagnostic costs should be also mentioned. They are very important, especially for the long-time horizon of analysis (see, e.g., [26] for an additional discussion).

Moreover, we assume that a value of the costs is related to a type of service (i.e., if it is a replacement or a repair), length of this service and a type of the connection. Therefore we have:

$$cost^{(j)}(t_i) = cost^{(j)}_{R,const} + cost^{(j)}_{R,Var}\left(RT_{t_i}\right),$$

where $cost^{(j)}(t_i)$ denotes a *total sum of costs* for the given *j*-th connection and time t_i, when a necessary service begins, $cost^{(j)}_{R,const}$ is a constant value (or a fixed cost, i.e., the value which is independent of length of a repair) and $cost^{(j)}_{R,Var}(.)$ is a variable cost of a repair (i.e., the value which is related to length of a repair). In the same manner, if for a replacement, we have:

$$cost^{(j)}(t_i) = cost^{(j)}_{P,const} + cost^{(j)}_{P,Var}\left(PT_{t_i}\right).$$

In the existing literature, the concept of a variable interest rate, which is used to estimate the present value (or the future value, see, e.g., [28]) of the maintenance costs, is still rarely used. But, as it was pointed out in [25, 26], the obtained results in the case of a variable rate significantly differ if they are compared to models with a constant yield. It is especially true if a long time horizon *T* (like 20 or even 50 years, which are quite common values for real-life WDSs) is taken into account. Then, to calculate the present value of the total sum of the costs of repairs and replacements, which is given by:

$$PV(cost) = \sum_{i,j} PV\left(cost^{(j)}(t_i)\right),$$

the one-factor Vasicek model (see, e.g., [8, 28]) is used to find a discounting factor $PV(.)$ for each respective cost $c^{(j)}(t_i)$. For this variable interest rate, a value of the interest rate r_t at time *t* is modelled by:

$$dr_t = a(b - r_t) + \sigma\, dW_t \ , \tag{9}$$

where W_t is the standard Brownian motion, *b* characterizes a long term mean level (i.e., the trajectory of r_t is directed to *b* during its long run), *a* reflects the speed of reversion towards *b*, and σ is instantaneous volatility (variability) of the trajectory related to the random component W_t. In the MC setting, an iterative scheme for a generation of increments Δr_t of the process (9) should be used (see, e.g., [8, 26] for a more detailed discussion and necessary formulas).

5. Imprecise setting of the model

As it was mentioned in Sec. 1, there are a few important cases, when our model can be described in an imprecise way. For example, data can be sparse or even unavailable, so to take into account opinions of the experts, the necessary parameters of the model are given as imprecise values (like, e.g., "the value of this parameter is about 5" or "this parameter is relatively low", see, e.g., [11] for an additional discussion). This impreciseness can be modelled using various types of fuzzy sets (see, e.g., [9, 11, 21, 26, 27, 28] for additional discussion) like, e.g., triangular fuzzy numbers (abbreviated further as TRFN), left-right fuzzy numbers (LRFN), trapezoidal fuzzy numbers (TPFN), interval-valued fuzzy numbers (IVFN), or using another approach, like shadowed sets (SHS). Of course, fuzzy or shadowed sets, which are applied in the considered setting, should be strictly related to available data and its interpretation.

In this paper, we further develop our previous works related to the fuzzy approach to describe a WDS (see [25, 26]). In the following, the whole model of the maintenance costs, together with the parameters of the introduced HRF, will be entirely fuzzified. Moreover, we also present a completely new approach, which is based on shadowed sets. Respective numerical algorithms for both these cases will be also provided.

5.1. Fuzzy approach

We start with some basic definitions and notation, which will be used in this paper. Additional details concerning the fuzzy approach can be found in, e.g., [7, 14, 28].

For a fuzzy subset $\tilde{A}$ of the set of real numbers *R*, we denote by $\mu_{\tilde{A}}$ its membership function $\mu_{\tilde{A}} : R \to [0,1]$ and by $\tilde{A}[\alpha] = \{x : \mu_{\tilde{A}}(x) \geq \alpha\}$ the α-level set (or the α-cut) of $\tilde{A}$ for $\alpha \in (0,1]$. Then $\tilde{A}[0]$ is the closure of the set $\{x : \mu_{\tilde{A}}(x) > 0\}$.

A fuzzy number $\tilde{a}$ is a fuzzy subset of *R* for which $\mu_{\tilde{A}}$ is a normal, upper-semicontinuous, fuzzy convex function with a compact support. Then for each $\alpha \in [0,1]$, the α-level set $\tilde{a}[\alpha]$ is a closed interval of the form $\tilde{a}[\alpha] = [a_L[\alpha], a_U[\alpha]]$, where $a_L[\alpha], a_U[\alpha] \in R$ and $a_L[\alpha] \leq a_U[\alpha]$.

A left-right fuzzy number (LRFN) is a fuzzy number with the membership function of the form:

$$\mu_{\tilde{a}}(x) = \begin{cases} L\left(\dfrac{x-a}{b-a}\right), & x \in [a,b] \\ 1, & x \in [b,c] \\ R\left(\dfrac{d-x}{d-c}\right), & x \in [c,d] \\ 0, & otherwise \end{cases},$$

where $L, R : [0,1] \to [0,1]$ are non-decreasing functions, such that $L(0) = R(0) = 0$ and $L(1) = R(1) = 1$. If both *L* and *R* are strictly linear functions, then this kind of LRFN is known as a trapezoidal fuzzy number (further abbreviated as TPFN), and we have:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a}{b-a},\ x\in[a,b]\\ 1,\ x\in[b,c]\\ \dfrac{d-x}{d-c},\ x\in[c,d]\\ 0,\ otherwise\end{cases}.$$

This kind of fuzzy number will be further denoted by $[a,b,c,d]$. Trapezoidal fuzzy numbers are one of the most commonly used types of fuzzy numbers (see, e.g., [9] for additional discussion). A triangular fuzzy number (TRFN) is a trapezoidal fuzzy number with its core reduced to a single value. Then its membership function has the form:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a}{b-a},\ x\in[a,b]\\ \dfrac{x-c}{b-a},\ x\in[b,c]\\ 0,\ otherwise\end{cases}.$$

TRFN will be further denoted by $[a,b,c]$. Some examples of the mentioned types of fuzzy numbers can be found in Fig. 4.

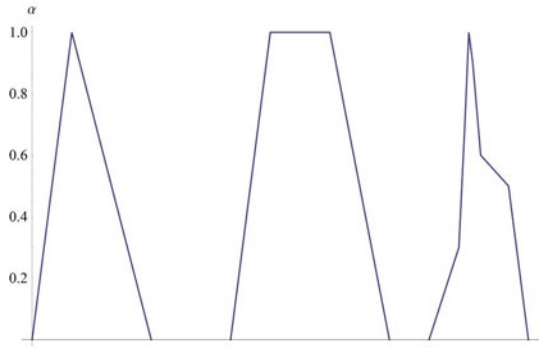


Fig.4. Examples of fuzzy numbers (the left-hand side to the right-hand side: a triangular fuzzy number, a trapezoidal fuzzy number, a left-right fuzzy number)

Let us assume, that our aim is to calculate the value of a function $f(x)$ for some parameter x, e.g., the present value of the maintenance costs $PV(c)$ depending on the constant cost of repair $c_{R,const}^{(j)}$. To approximate a fuzzy value $\tilde{f}(\tilde{x})$ for a fuzzy parameter $\tilde{x}$, two-step procedure (see Algorithm 3) is conducted. During the first step, monotonicity of $f(x)$ is checked and a specified value $\alpha\in[0,1]$ is set. If $f(x)$ is a non-decreasing function, then for the given α, the left endpoint $f_L(\tilde{x})[\alpha]$ of the respective α-level set $\tilde{f}(\tilde{x})[\alpha]$ is approximated using the crisp value $x_L[\alpha]$, i.e., $f_L(\tilde{x})[\alpha]=f(x_L[\alpha])$. And the same applies to the right endpoint $f_U(\tilde{x})[\alpha]$ and $x_U[\alpha]$, i.e., $f_U(\tilde{x})[\alpha]=f(x_U[\alpha])$. In contrary, if $f(x)$ is a non-increasing function, first $x_U[\alpha]$, then $x_L[\alpha]$ should be used to evaluate the respective α-cut, which is given by $[f_L(\tilde{x})[\alpha],f_U(\tilde{x})[\alpha]]$. Then the same step is repeated for other values of α. Usually, we start from $\alpha=0$ and end at $\alpha=1$ with some fixed increment $\Delta\alpha>0$. During the second step, the whole fuzzy number $\tilde{f}(\tilde{x})$ is constructed, based on the α-level sets which were previously calculated. The "missing" α-level sets are directly approximated using respective linear segments between the known α-cuts (see, e.g., [20, 26, 28] for further discussion). Of course, more complex functions (e.g., polynomials) can be also applied, but usually simple linear functions are sufficient. Let us illustrate the above procedure with a simple example:

Example. Let us suppose, that we are interested in an approximation of the maintenance costs $PV(c)$ for a fuzzy value of the constant cost of repair $\tilde{c}_{R,const}^{(j)}$, i.e., we would like to find $\widetilde{PV}\left(\tilde{c}_{R,const}^{(j)}\right)$. As it is easily seen, the respective function is an increasing one in this case. Therefore, to find $\widetilde{PV_L}\left(\tilde{c}_{R,const}^{(j)}\right)$ (or $\widetilde{PV_U}\left(\tilde{c}_{R,const}^{(j)}\right)$, respectively) for the given α, the value $\tilde{c}_{R,const,L}^{(j)}$ (or $\tilde{c}_{R,const,U}^{(j)}$, respectively) should be applied. And we can start our approximation of the fuzzy output using $\alpha=0$ with an increment $\Delta\alpha=0.1$ up to $\alpha=1$.

Algorithm 3 (Approximation of the fuzzy output)

Input: A function $f(x)$, an increment $\Delta\alpha>0$.

Output: An approximation of $\tilde{f}(\tilde{x})$.

$\alpha=0$

while $\alpha\le1$ **do**

Check monotonicity of $f(x)$*;*

Calculate $[f_L(\tilde{x})[\alpha],f_U(\tilde{x})[\alpha]]$ *using* $[f(x_L[\alpha]),f(x_U[\alpha])]$ *(if* $f(x)$ *is a non-decreasing function) or* $[f(x_U[\alpha]),f(x_L[\alpha])]$ *(otherwise);*

$\alpha=\alpha+\Delta\alpha$ *;*

Approximate missing values $[f_L(\tilde{x})[\alpha],f_U(\tilde{x})[\alpha]]$ *using linear segments;*

return $\tilde{f}(\tilde{x})$

Because in the following we focus on the present value of the maintenance costs and its fuzzy counterpart, monotonicity of $PV(c)$ depending on the considered parameters is summarized in Table 1, where a plus sign denotes non-decreasing and a minus sign – a non-increasing function of the given parameter. However, a similar table can be prepared for other kinds of the desired fuzzy output.

Table 1. Monotonicity of $PV(c)$ *depending on different parameters of the model*

Parameter	a_0	a_1	a_2	b_0	x_0^*	x_1^*	α_r	c_R,c_P	d_R,d_P	Variable and constant costs
Monotonicity	-	+	+	+	-	-	+	-	-	+

5.2. Approach based on shadowed sets

We start with some basic definitions and notation, which will be used in this paper further on. Additional details concerning shadowed sets can be found in, e.g., [10, 21].

A shadowed set (SHS, for short) $\dot{S}$ in a universe of discourse X is a set-valued mapping $\dot{S}: X \to \{0,[0,1],1\}$ having the following interpretation (see, e.g., [21]):

- all elements of X for which $\dot{S}(x)=1$ are called a *core* of the shadowed set $\dot{S}$ and they embrace all elements that are fully compatible with the concepts conveyed by $\dot{S}$,
- all elements of X for which $\dot{S}(x)=0$ are completely excluded from the concept described by $\dot{S}$,
- all elements of X for which $\dot{S}(x)=[0,1]$, called *a shadow*, are uncertain.

Then, for a shadowed set $\dot{S}$ we have its core (defined by $\{x \in X, \dot{S}(x)=1\}$), the shadow $\{x \in X, \dot{S}(x)=[0,1]\}$ and the support $cl\{x \in X, \dot{S}(x) \neq 0\}$. The usage of the unit interval for the shadow shows that any element from the shadow could be excluded or exhibit partial membership or could be fully allocated to $\dot{S}$ (see, e.g., [10]).

In the following, we consider shadowed sets defined for real values only, i.e. when $X=R$ (see Fig. 5 for the respective example). Then, a shadowed set will be denoted by $[s_1,s_2,s_3,s_4]_{SH}$, where its core is given by the interval $[s_2,s_3]$, its shadow by $(s_1,s_2)\cup(s_3,s_4)$ and its support by $[s_1,s_4]$. Shadowed sets are conceptually close to rough sets (see, e.g., [21]), i.e. their cores can be treated as the regions whose elements belong to the concept under discussion, their shadows – the regions, where the membership grade is doubtful, and the outer parts – the regions whose elements definitely do not belong to the concept.

Fig. 5. Example of a shadowed set

There exist important links between concepts of a fuzzy set and a shadowed set. First, a fuzzy set can be approximated using a shadowed set. Based on the initial fuzzy set, a corresponding shadowed set, that at the same time captures "the essence" of this fuzzy set, reduces computational efforts related to a membership function, and simplifies the interpretation, can be constructed. The idea behind this approach was discussed in [21]. As he noted "we are usually far more confident about assigning values close 1 (thus counting the elements in) or 0 (therefore making the corresponding element excluded from the concept). On the other hand, the membership values (such as those around 0.5) always spark some hesitation and are always more difficult to place on a simple numeric scale". Therefore, a shadowed set is constructed (induced) from the initial fuzzy set with an elevation of some membership values ("close to 1" or "high enough") and with a reduction of others (which are "close to 0" or "low enough"). Then, the necessary computational effort related to using the obtained shadowed set (instead of its fuzzy counterpart) is reduced, because only two "cuts" (instead of the whole [0,1] interval) are used in further calculations. Because this procedure can reduce vagueness, some additional restrictions are taken into account to maintain its overall value.

In [10], a respective approximation procedure, which is related to the optimization of two weighting functions, is introduced. Then, a TPFN, which is given by $[a,b,c,d]$, can be approximated by a SHS $[s_1,s_2,s_3,s_4]_{SH}$ using formulas

$$s_1=\frac{2}{3}a+\frac{b}{3}, s_2=\frac{a}{3}+\frac{2}{3}b, s_3=\frac{2}{3}c+\frac{d}{3}, s_4=\frac{c}{3}+\frac{2}{3}d. \quad (10)$$

Example. Let us suppose, that our TPFN is described by values $[1,3,4,7]$. Then, from (10) we get $[1.6667,2.3333,5,6]_{SH}$.

Second, calculation of the value of a function, which is related to a shadowed set, can be seen as a simplified approach for a fuzzy set, or – stated in another way – based on interval calculations. Namely, to find a value of a function $\dot{f}(\dot{S})$ for some shadowed set $\dot{S}$ we have to take into account only two intervals $[s_1,s_4]$ and $[s_2,s_3]$ similarly as for a fuzzy set (but with two α-level sets only, i.e. when α=0 and α=1). Then, Algorithm 3 can be directly modified if parameters of the considered model are given with shadowed sets instead of fuzzy sets. This leads us to Algorithm 4.

Algorithm 4 (Approximation of the shadowed set output)

Input: A function $f(x)$.

Output: An approximation of $\dot{f}(\dot{x})$.

Check monotonicity of $f(x)$*;*

Calculate the core of $\dot{f}$ *using* $[f(s_3),f(s_4)]$ *(if* $f(x)$ *is a non-decreasing function) or* $[f(s_4),f(s_3)]$ *(otherwise);*

Calculate the shadow of $\dot{f}$ *using* $(s_1,s_2)\cup(s_3,s_4)$ *(if* $f(x)$ *is a non-decreasing function) or* $(s_4,s_3)\cup(s_2,s_1)$ *(otherwise);*

return $\dot{f}(\dot{x})$

6. Examples of numerical simulations

After providing the necessary algorithms, we present some examples based on the Monte Carlo simulations. In the following, numerical approximations of the present value of the maintenance costs for an exemplary WDS for both the fuzzy and the shadowed sets settings are discussed. The applied parameters of the considered model can be divided into four groups:

1. parameters of the given type of the connection, which are related to the introduced HRF (1), namely $a_0,a_1,a_2,b_0,x_0^*,x_1^*,\alpha_r,n_r$,
2. parameters, which depend on the respective connection, and are related to the maintenance costs $cost_{R,const}, cost_{P,const}, cost_{R,var}(.), cost_{P,var}(.)$ or the lengths of times of necessary services (i.e. repairs and replacements), like parameters of the random distributions for RT_i and PT_i,
3. parameters of the interest rate model, which are related to the one-factor Vasicek model (6), i.e. r_0,a,b,σ,
4. other parameters, like P^* and time range for the whole simulation T.

6.1. Numerical analysis for the fuzzy setting

We assume that all the parameters related to the considered HRF (1), the costs of the repairs and the replacements, and the length of the services are fuzzified, i.e., they are given as triangular or trapezoidal fuzzy numbers. Only the parameters of the one-factor Vasicek model and the considered times (i.e. P^* and T) are described by crisp (real) values. Time is measured in years.

In the following, we partially use fuzzified versions of parameters considered in [26] (see Table 2). As noted in Sect. 2 and Sect. 3, the respective expected value for the period between malfunctions is "about 4.5 years" and for the time of repair is "about 1.68 days".

For example, if $a_0 = [0.5, 0.6, 0.65, 0.7]$, then the directional component of the descending linear part of (1) is described by a TPFN for which its core is given by the interval $[0.6, 0.65]$ and its support by $[0.5, 0.7]$. Therefore, we are completely sure, that the considered value is in the interval $[0.6, 0.65]$ and also "certain to some extent", that this value is in $[0.5, 0.7]$, so this parameter is "about 0.6.0–65 plus 0.05 /minus 0.1" (for additional remarks concerning the fuzzy scales and their interpretation, see, e.g., [15]). And for $cost_{R,const} = [0.5, 1, 2]$ we can say, that the constant costs of a single repair are "about 1, minus 0.5 (50% of the core value) / plus 1 (100% of the core value)", so this parameter has longer right-hand support. In the same manner, if $d_R = [0.012, 0.014, 0.016]$, then the maximum time of a repair is "about 5 days". The unconditional replacement age P^* is equal to 5 years, and the time range of the simulations is equal to 50 years, which is a value commonly used in real-life applications for other WDS.

In this example, we use only triangular or trapezoidal fuzzy values of the parameters, but more complex types of fuzzy numbers can be also applied (like, e.g., LRFNs). However, these two types are the most commonly spotted in real-life applications because of their simple description, together with easy and direct interpretation.

To find fuzzy approximations of the desired output, one million Monte Carlo simulations for 10 pipes (which are identical, i.e. they have the same parameters) were conducted with the increment $\Delta\alpha = 0.1$. Using the rather low-end hardware (i5-7400 3 GHz, 8 GB RAM, Win 7 Pro) as for the modern standards and C++ (Visual Studio 2019), the whole simulation procedure took about 1 hour.

Some examples of the obtained output were summarized in Fig. 6–9. In Fig. 6 we can find fuzzy approximations of the minimum, the mean and the maximum of the costs of the single repair. As we can see, these values are described by a TRFN (the minimum) or TPFNs (the mean and the maximum), but the mean (given by $[0.766419, 1.32643, 1.37307, 2.39986]$) has the rather narrow core, mainly due to the existing discounting factor. Similar measures for the costs of the single replacement are given in Fig. 7. Moreover, fuzzy approximations of the means for the costs of the repairs and the replacements differ substantially, the first one is lower than the second (which is equal to $[4.79961, 6.83968, 6.953, 8.95973]$), and they have other kinds of skewness. Their relatively wide supports should be also noticed.

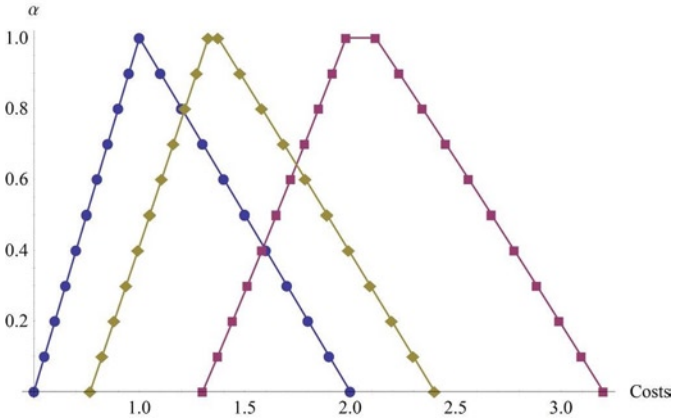


Fig. 6. Fuzzy approximations of the minimum (circles), the mean (diamonds), and the maximum (rectangles) of the repairs costs

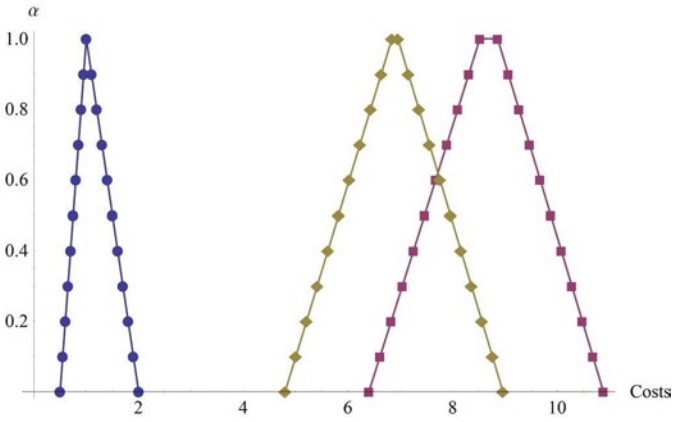


Fig. 7. Fuzzy approximations of the minimum (circles), the mean (diamonds), and the maximum (rectangles) of the replacements costs

Table 2. Fuzzy and crisp parameters applied in exemplary numerical simulations

Parameter	a_0	a_1	a_2	b_0
Value	$[0.5, 0.6, 0.65, 0.7]$	$[0.1, 0.2, 0.3]$	$[0.7, 0.8, 0.9, 1]$	$[0.6, 0.65, 0.7]$

Parameter	α_r	x_0^*	x_1^*
Value	$[0.05, 0.1, 0.15]$	$[0.4, 0.5, 0.6, 0.7]$	$[9, 10, 11, 12]$

Parameter	c_R	d_R	c_P	d_P
Value	$[8, 10, 11, 12]$	$[0.012, 0.014, 0.016]$	$[4, 5, 6]$	$[0.024, 0.026, 0.028, 0.03]$

Parameter	$cost_{R,const}$	$cost_{R,var}$	$cost_{P,const}$	$cost_{P,var}$
Value	$[0.5, 1, 2]$	$[50, 70, 80, 100]$	$[4, 6, 8]$	$[80, 90, 110, 120]$

Parameter	a	b	r_0	σ	P^*	T
Value	0.1	0.05	0.04	0.001	5	50

Another output is shown in Fig. 8, where the fuzzy approximation of the average number of the repairs $\overline{x}_R$ of the whole WDS is plotted. The obtained LRFN is very close to a TPFN, but clearly differs

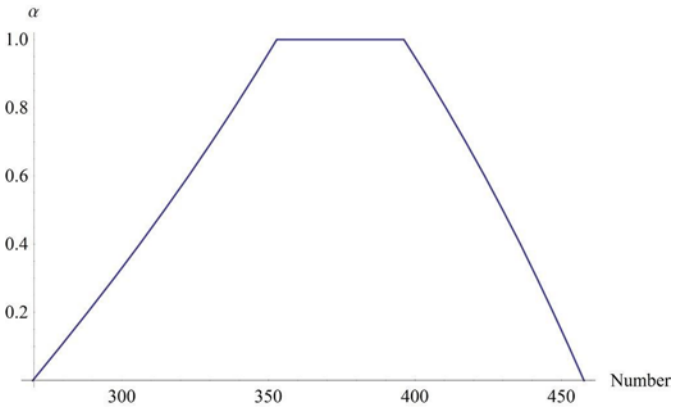
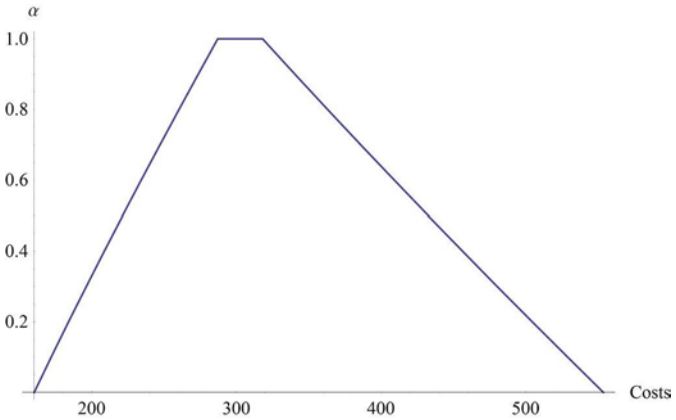


Fig. 8. Fuzzy approximation of the average number of repairs $\overline{x}_R$

from this kind of fuzzy number because of the visible curvatures of its membership function for the left- and the right-hand sides. Moreover, its support, which is equal to $[154.314, 349.395]$, is rather wide, then the obtained impreciseness is rather high in this case.

The most important simulation result, i.e. fuzzy approximation of the present value of the whole maintenance costs $PV(cost)$, is presented in Fig. 9. In this case, we have a fuzzy number that may be identified with a TPFN, because no curvatures are clearly visible. The obtained fuzzy number has the longer right-hand support (so we can expect some costs "on plus" rather than "on minus"), with the core given by the interval $[359.268, 384.822]$ and the rather wide support, which is equal to $[209.44, 621.271]$. Once more, the obtained impreciseness for $\alpha = 0$ is rather high. Clearly, for the given other value of α, we can find the respective α-level set of $PV(cost)$, which approximates the result as the interval.

Fig. 9. Fuzzy approximation of the present value of the maintenance costs $PV(cost)$



It may be fruitful to compare the simulation results if one (or more) of the important parameters are changed. The whole optimization procedure can be even applied to minimize the whole maintenance costs (see, e.g., [26]). In the following, instead of $P^* = 5$, higher value of the unconditional replacement age, which is equal to 20 years (i.e. $P^* = 20$) is assumed. Some of the obtained results can be found in Fig. 10–11.

As we can see in Fig. 11, for the longer unconditional replacement age, the present value of the maintenance costs is highly reduced (to a TPFN given by $[72.4155, 120.987, 134.428, 222.912]$, so even more than 65% for some α-cuts). Moreover, the average number of repairs

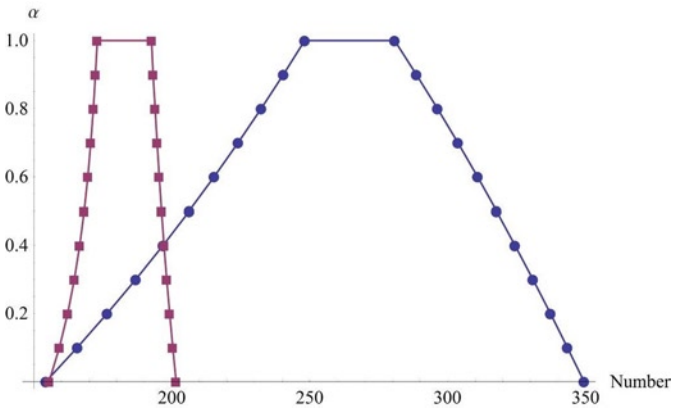


Fig. 10. Fuzzy approximation of the average number of repairs $\overline{x}_R$ *for* $P^* = 5$ *(years, circles) and* $P^* = 20$ *(years, rectangles)*

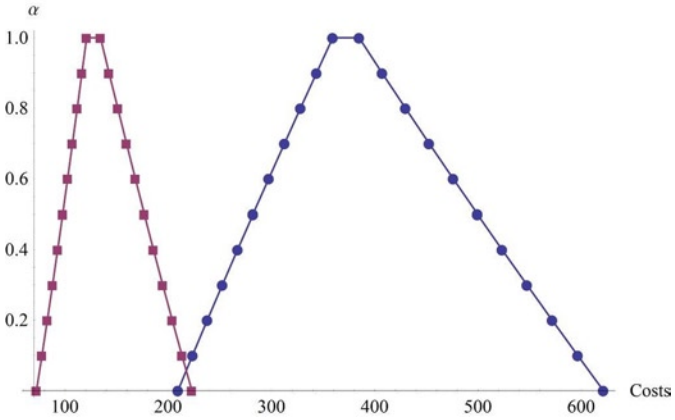


Fig. 11. Fuzzy approximation of the present value of the maintenance costs $PV(cost)$ *for* $P^* = 5$ *(years, circles) and* $P^* = 20$ *(years, rectangles)*

(see Fig. 10) is also significantly lower, with the highly reduced length of its support (hence, its impreciseness, too).

6.2. Numerical analysis in the case of the shadowed sets

As it was noticed in Sec. 1, it may be fruitful to use a concept of shadowed sets instead of fuzzy numbers. First, it may be easier for an expert to describe a considered parameter as a set of four values only, rather than formulate a more complex opinion concerning a whole course of a membership function. Second, calculations related to shadowed sets are usually easier, if they are compared with numerical efforts required for fuzzy numbers, because only two levels are necessary to obtain the desired output instead of, e.g., twenty α-level sets. Hence, only two simulation runs are required instead of twenty of them. But the obtained shadowed set still reflects some vagueness, which may be expressed as an imprecise opinion of an expert.

In the following, to compare the simulated results for both approaches, we approximate all fuzzy parameters (see Table 2) using shadowed sets and the formulas (10), e.g., we have:

$$a_0 = \left[0.5(3), 0.5(6), 0.6(6), 06.8(3)\right]_{SH},$$
$$a_1 = \left[0.1(3), 0.1(6), 0.2(3), 0.2(6)\right]_{SH},$$
$$a_2 = \left[0.7(3), 0.7(6), 0.9(3), 0.9(6)\right]_{SH},$$
$$b_0 = \left[0.61(6), 0.7(6), 0.9(3), 0.9(6)\right]_{SH}.$$

Values of other parameters can be easily computed using (10).

The simulated mean of the costs for single repair is plotted in Fig. 12, the mean of the costs for a single replacement in Fig. 13, the average number of repairs $\overline{x}_R$ in Fig. 14 and the present value of the main-

tenance costs $PV(c)$ in Fig. 15. If they are compared with their fuzzy counterparts, wider cores and narrower shadows are clearly visible. Therefore, the simulated shadowed sets are different, but still similar to the previously obtained fuzzy sets, e.g., now the mean of the single repair costs is equal to $[0.956047, 1.14272, 1.71829, 2.06055]_{SH}$. Generally, the areas of values, which are "completely sure", are wider and the areas, which are "doubtful", are narrower. But their "shapes" (i.e. when the left- or the right-hand shadow is wider in comparison with its right- or left-hand counterpart) are similar to the respective fuzzy outputs. It is especially clearly visible in the case of $PV(c)$ (compare Fig. 9 with Fig. 15).

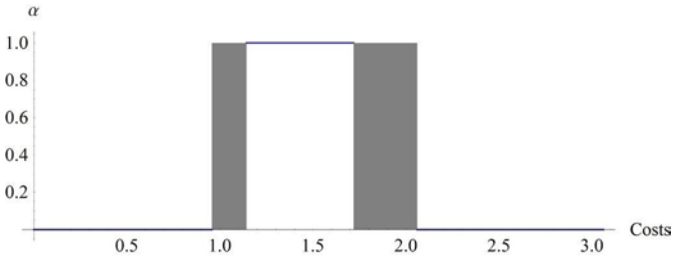


Fig. 12. Shadowed set approximation of the mean of the repairs costs

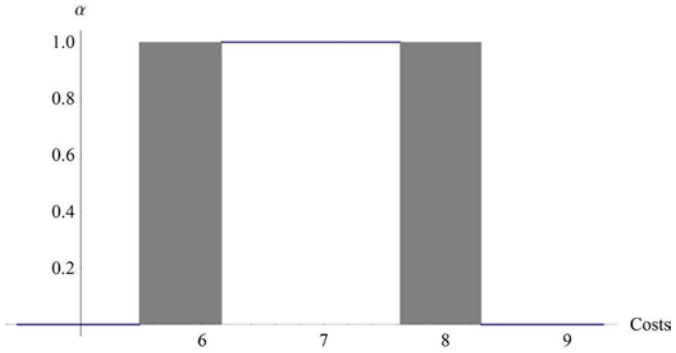


Fig. 13. Shadowed set approximation of the mean of the replacements costs

7. Conclusion

In this paper, a new kind of a hazard rate function for the time between malfunctions of a pipeline is proposed. This HRF is a U-shaped function, which also depends on the number of the previous repairs of the given connection. The numerically efficient simulation algorithm for this HRF is also provided. Additionally, times of the maintenance services (i.e. repairs and replacements) are modelled using the random distribution with decreasing intensity and finite support. The introduced models are then used to approximate the present value of the maintenance costs and other important characteristics for a WDS in two imprecise settings based on fuzzy numbers and shadowed sets. The general framework for these two imprecise setups together with the respective numerical algorithms are also discussed. These two approaches lead us to better incorporation of the experts' knowledge and a more proper, closer to real-life modelling of imprecise parameters of the considered model. To calculate the present value of the maintenance costs, the one-factor Vasicek model is used, as it doesn't incorporate error appearing in the case of a constant interest rate. The numerical examples of the maintenance costs and other important characteristics for a WDS are also analysed.

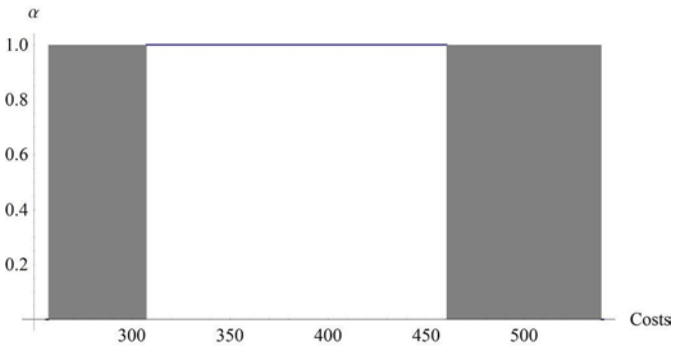


Fig. 14. Shadowed set approximation of the average number of repairs $\overline{x}_R$.

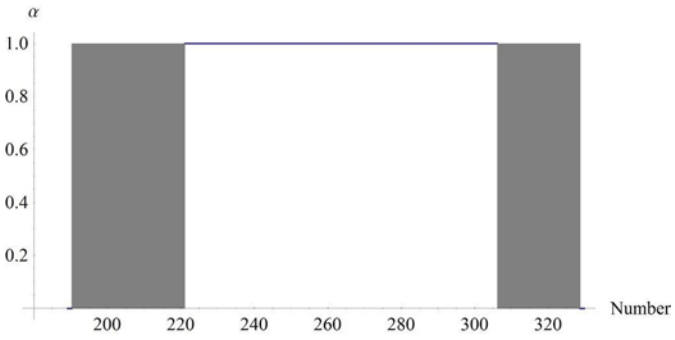


Fig. 15. Shadowed set approximation of the present value of the maintenance costs $PV(cost)$.

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